2018-10-24-1

Last Time: $W_m = \int_0^{\lambda} i \, d\lambda^*$ $\qquad f^e = -\frac{\partial W_m}{\partial x}$ $\qquad i = \frac{\partial W_m}{\partial \lambda}$

Today: use i, x as independent variables

$$\frac{dW_m}{dt} = i\frac{d\lambda}{dt} - f^e \frac{dx}{dt}$$

$$\frac{d(i\lambda)}{dt} = i\frac{d\lambda}{dt} + \lambda\frac{di}{dt} \quad \Rightarrow \quad i\frac{d\lambda}{dt} = \frac{d(i\lambda)}{dt} - \lambda\frac{di}{dt}$$

$$\frac{dW_m}{dt} = \frac{d(i\lambda)}{dt} - \lambda\frac{di}{dt} - f^e\frac{dx}{dt} \quad \Rightarrow \quad \frac{d}{dt}(W_m - i\lambda) = -\lambda\frac{di}{dt} - f^e\frac{dx}{dt}$$

$$\frac{d}{dt}(i\lambda - W_m) = \lambda\frac{di}{dt} + f^e\frac{dx}{dt}$$

define co-energy: $W_m' = i\lambda - W_m$

$$\boxed{\frac{dW_m'}{dt} = \lambda\frac{di}{dt} + f^e\frac{dx}{dt}}$$

i and x are independent variables!

$$\frac{dW_m'}{dt} = \frac{\partial W_m'}{\partial i}\frac{di}{dt} + \frac{\partial W_m'}{\partial x}\frac{dx}{dt}$$

$$\boxed{\lambda = \frac{\partial W_m'}{\partial i} \qquad f^e = \frac{\partial W_m'}{\partial x}}$$

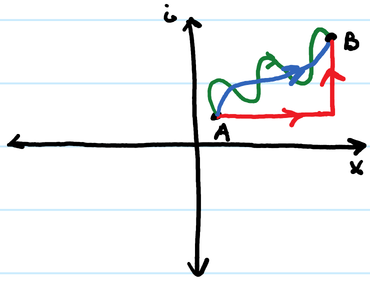
* So, like before once the co-energy is known, f^e can be found by differenciating.

2018-10-24-2

$$\frac{dW_m'}{dt} = \lambda \frac{di}{dt} + f^e \frac{dx}{dt} \Rightarrow dW_m' = \lambda di + f^e dx$$

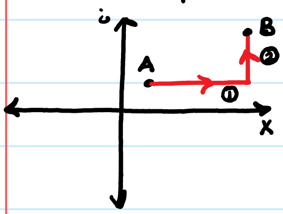
$$\int_{W_{m0}'}^{W_m'} dW_m'^* = \int_{i_0}^{i} \lambda di^* + \int_{x_0}^{x} f^e dx^*$$

$$W_m' - W_{m0}' = \int_{i_0}^{i} \lambda di^* + \int_{x_0}^{x} f^e dx^*$$



* Again, conservative field: path independence

* Select path for most easy integration.



* set $i_0 = 0$ and say $f^e(i=0) = 0$

* on ①: $i = 0$ = constant

$$\int_0^0 \lambda di^* = 0 \qquad \int_{x_0}^{x} f^e dx^* = 0$$

* on ②

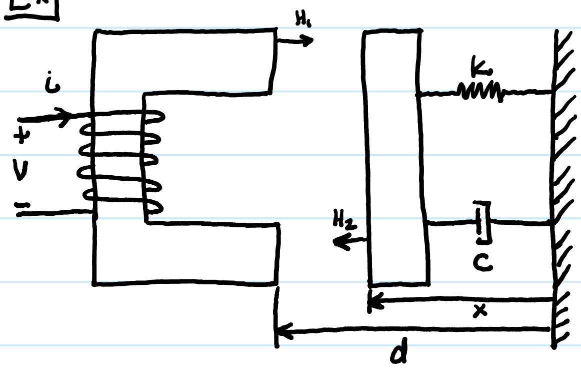
$$\int_0^{i} \lambda di^* \neq 0 \qquad \int_x^x f^e dx = 0$$

$$\Rightarrow \boxed{W_m' = \int_0^i \lambda di^* \qquad f^e = \frac{\partial W_m'}{\partial x}}$$

if $\lambda = L(x) i \Rightarrow W_m' = \int_0^i L(x) i^* di^* \Rightarrow W_m' = \frac{1}{2} L(x) i^2$

$$f^e = \frac{\partial W_m'}{\partial x} \Rightarrow \boxed{f^e = \frac{1}{2} \frac{\partial L}{\partial x} i^2}$$

Ex



$$\boxed{\phi = \left[\frac{\mu_0 A N}{2(d-x)}\right] i}$$

$$\lambda = N\phi \Rightarrow \lambda = \left[\frac{\mu_0 A N^2}{2(d-x)}\right] i$$

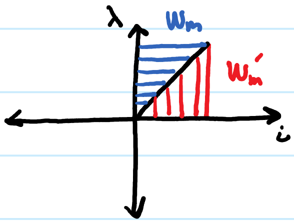
$$W_m' = \int_0^i \left[\frac{\mu_0 A N^2}{2(d-x)}\right] i^* di^* \Rightarrow W_m' = \frac{1}{2}\left[\frac{\mu_0 A N^2}{2(d-x)}\right] i^2$$

$$f^e = \frac{\partial W_m'}{\partial x} \Rightarrow \boxed{f^e = \frac{1}{2}\left[\frac{\mu_0 A N^2}{2(d-x)^2}\right] i^2 = \frac{1}{2}\left[\frac{2}{\mu_0 N^2 A}\right]\lambda^2}$$

* Same as when using W_m, as expected!

Energy vs. Coenergy

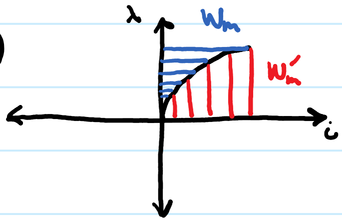
$W_m = \int_0^\lambda i\, d\lambda^*$ $\qquad$ $W_m' = \int_0^i \lambda\, di^*$



$W_m + W_m' = \lambda i$ as expected

if $\lambda = L(x) i$ (linear) $\Rightarrow W_m = W_m'$

$\lambda = f(i, x)$ (non-linear)



$W_m + W_m' = \lambda i$ as expected

$W_m \neq W_m'$

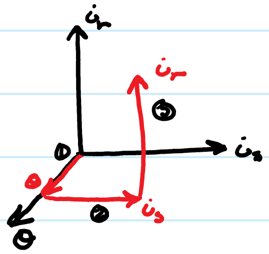
Multiple input system: For example, rotor-stator system

$$\lambda_s = L_s i_s + L_m(\theta) i_r$$
$$\lambda_r = L_m(\theta) i_s + L_r i_r$$

$$\frac{dW_m'}{dt} = \lambda_s \frac{di_s}{dt} + \lambda_r \frac{di_r}{dt} + T^e \frac{d\theta}{dt}$$

$$\Rightarrow W_m' = \int_{i_{s0}}^{i_s} \lambda_s \, d\hat{i}_s + \int_{i_{r0}}^{i_r} \lambda_r \, d\hat{i}_r + \int_{\theta_0}^{\theta} T^e \, d\hat{\theta}$$

Path:



$$i_{r0} = i_{s0} = 0$$
$$T^e(i_r=0, i_s=0) = 0$$

on ①: $\int_0^0 \lambda_s \, d\hat{i}_s = \int_0^0 \lambda_r \, d\hat{i}_r = 0$ $\qquad \int_{\theta_0}^{\theta} 0 \, d\theta = 0$

on ②: $\int_0^{i_s} \lambda_s(\hat{i}_s, 0, \theta) \, d\hat{i}_s \neq 0$ $\qquad \int_0^0 \lambda_r \, d\hat{i}_r = \int_\theta^\theta T^e \, d\hat{\theta} = 0$

on ③: $\int_0^{i_r} \lambda_r(i_s, \hat{i}_r, \theta) \, di_r \neq 0$ $\qquad \int_{i_s}^{i_s} \lambda_s \, d\hat{i}_s = \int_\theta^\theta T^e \, d\hat{\theta} = 0$

$$W_m' = \int_0^{i_s} \lambda_s(\hat{i}_s, 0, \theta) \, d\hat{i}_s + \int_0^{i_r} \lambda_r(i_s, \hat{i}_r, \theta) \, d\hat{i}_r$$

$$T^e = \frac{\partial W_m'}{\partial \theta}$$

$$\Rightarrow \boxed{\begin{aligned} W_m' &= \frac{1}{2} L_s i_s^2 + L_m(\theta) i_r i_s + \frac{1}{2} L_r i_r^2 \\ T^e &= \frac{\partial L_m}{\partial \theta} i_r i_s \end{aligned}}$$